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INFRARED ASYMPTOTICS AND DAYSON-SCHWINGER EQUATIONS FOR THE GAUGE-INVARIANT SPINOR GREEN FUNCTION IN QUANTUM ELECTRODYNAMICS

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The Dayson-Schwinger equations for the gauge-invariant (G.I.) spinor Green function are derived for an Abelian case. On the basis of these equations as well as the functional integration method the behaviour of the G.I. spinor propagator is studied in the infrared region. It is shown that the G.I. propagator has a singularity of a simple pole in this region.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Инфракрасная асимптотика и уравнения Дайсона-Швингера для калибровочно-инвариантной спинорной функции Грина в квантовой электродинамике

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Получены уравнения Дайсона-Швингера для калибровочно-инвариантной спинорной функции Грина в абелевом случае. На основе этих уравнений, а также с помощью функциональных методов изучается поведение калибровочно-инвариантного спинорного пропагатора в инфракрасной области. Показано, что введенный таким образом пропагатор имеет в этой области особенность в виде простого полюса.

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During the last few years there appeared a lot of papers where the behaviour of the Green functions in the infrared region was studied (see for instance[1,2] and references therein). The interest in this problem stems from a widely discussed possibility of the relation of the infrared asymptotics of the QCD Green functions with the problem of quark confinement.

In this article we shall study the infrared behaviour on the basis of the gauge-invariant (G.I.) variables

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which have been introduced in our previous papers [3]. We shall start with the definition of the G.I. Green function

$$G(x,y) = i\langle 0|T\psi(x)P\exp[ig\int_x^y d\xi^\mu A_\mu(\xi)]\bar{\psi}(y)|0\rangle . \qquad (1)$$

Our first aim is to find what form would take an analog of the Dayson-Schwinger equation for (1).

Previously the propagator (1) was considered in papers [4] for the purpose of a gauge-invariant definition of quark mass and studied in [5] in the framework of exactly solvable Schwinger and Bloch-Nordsick models. This function was also considered in [6] and calculated to the second order of perturbation theory in the massless case [7].

As for the integration contour in the P-ordered exponent in (1) we choose a piece of the straight line that connects the points x and y (see [4-7]). By applying the well-known method of derivation of the Schwinger equation in terms of functional derivatives for the gauge-dependent spinor propagator [2], we find that the G.I. equation for the propagator (1) in the presence of the vector source J_μ has the form*

$$[i\gamma_\mu(\frac{\partial}{\partial x_\mu} - g\frac{\delta}{\delta J_\mu(x)} - g[\frac{\partial}{\partial x_\mu}\int_x^y d\xi^\nu \frac{\delta}{\delta J^\nu(\xi)}] - igu^\mu(x) -$$

$$- ig[\frac{\partial}{\partial x_\mu}\int_x^y d\xi^\nu u_\nu(\xi)]) - m]G(x,y|J) = -\delta(x-y). \qquad (2)$$

The second equation has the standard form

$$u_\mu(x) = \int dz D^0_{\mu\nu}(x,z)[J^\nu(z) + igSp(\gamma^\nu G(z,z/J))]. \qquad (3)$$

where $D^0_{\mu\nu}$ is the vector field propagator.

It can be shown that the Dayson equation for the G.I. Green function (1) has the form

$$[i\hat{\partial}_x - m + g\hat{B}(x|y)G(x,y|u) - \int dy' M(x,y'|u)G(y',y|u) = -\delta(x-y), \qquad (4)$$

where $\hat{B}(x|y)$ is the G.I. vector field defined by the formula

$$B_\mu(x|y) = u_\mu(x) + \frac{\partial}{\partial x^\mu}\int_x^y d\xi^\nu u_\nu(\xi) \qquad (5)$$

and $M(x,y|u)$ is the G.I. mass operator defined as follows

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* Our notations coincides with this of [2].

$$M(x,y|u) = ig\gamma^{\mu}\int dy' \langle 0|T\psi(x)\exp[ig\int_x^{y'} d\xi^{\nu}A_{\nu}(\xi)]\times$$

$$\times\bar{\psi}(y')B_{\mu}(x|y')|0\rangle G^{-1}(y',y|u). \tag{6}$$

In order to find the asymptotics of the Green function (1) in the infrared region we shall make use of Eq.(2) and apply the method suggested in /8/. With our choice of the integration contour we pass to the functional variable u_{μ} and putting $y = 0$ find from (2) the equation

$$\{i\gamma^{\mu}[\frac{\partial}{\partial x^{\mu}} + g(\hat{\phi}_{\mu}(x) - \frac{\partial}{\partial x^{\mu}}x^{\nu}\int_0^1 ds\,\hat{\phi}_{\nu}(sx)) -$$

$$-ig(u_{\mu}(x) - (\frac{\partial}{\partial x^{\mu}}x^{\nu}\int_0^1 ds\,u_{\nu}(sx)))] - m\}G(x|u) = -\delta(x), \tag{7}$$

where $\hat{\phi}_{\mu}(x) = \int dy D_{\mu\nu}(x-y)\frac{\delta}{\delta u_{\nu}(y)}$.

Now we shall pass to the momentum representation and consider the expression

$$\Phi_{\mu}(p|u) = \int dx e^{ipx}[\hat{\phi}_{\mu}(x) - \frac{\partial}{\partial x^{\mu}}x^{\nu}\int ds\,\hat{\phi}_{\nu}(sx)]G(x|u). \tag{8}$$

Expression (8) can be rewritten in another form (we shall denote the Fourier transform of $\hat{\phi}_{\mu}(x)$ as $\hat{\phi}_{\mu}(k)$)

$$\Phi_{\mu}(p|u) = \int_0^1 ds\int\frac{dk}{(2\pi)^4}[\hat{\phi}_{\mu}(k)G(p-k|u) + k_{\mu}\frac{\partial\hat{\phi}_{\nu}(k)}{\partial k_{\nu}}G(p-sk|u)]. \tag{9}$$

In accordance with /8/ one can neglect the dependence of $G(p-k|u)$ and $G(p-sk|u)$ in (9) on the value of k. In this case the integrand in (9) becomes a 4-divergence of $k_{\mu}\hat{\phi}_{\nu}(k)$ and therefore we find in the infrared limit that $\Phi_{\mu}(p|u) = 0$.

In the analogous way it can be shown that in the infrared limit

$$\int dx e^{ipx}[u_{\mu}(x) - \frac{\partial}{\partial x^{\mu}}x^{\nu}\int_0^1 ds\,u_{\nu}(sx)]\,G(x|u) \simeq 0. \tag{10}$$

Thus the G.I. Green function $G(p)$ in the region $p^2 = m^2$ obeys the equation $(\hat{p}-m)G(p) = -1$ and therefore in infrared limit it has a singularity of a simple pole

$$G(p) = \frac{1}{m-\hat{p}} \quad \text{as} \quad p^2 \simeq m^2. \tag{11}$$

Supposing that the counterterms for the renormalization of the G.I. Green function have the same structure as in the case of the renormalization of the S-matrix, we find that the renormalized function $G'(p)$ in the infrared limit has a simple pole $G'(p) \simeq z_2^{-1}/(m-\hat{p})$ as well.

Let us study the infrared asymptotics of (1) on the basis of the method of functional integration [9]. After the integration over the fermion fields we shall represent the vector fields A_μ

$$G(x, y) = \int DA \frac{\det[i\hat{\partial} + g\hat{A} - m]}{\det[i\hat{\partial} - m]} \exp\{iS_0[A]\} \times$$
$$\times G(x, y|A) \exp[ig \int_x^y d\xi^\mu A_\mu(\xi)], \tag{12}$$

where $S_0[A]$ is a free action of the vector field and $G(x,y|A)$ is the Green function of the fermion in an external field A_μ. The function $G(x,y|A)$ can be written in the form of functional integral [9]

$$G(x, y|A) = i[i\hat{\partial}_x + g\hat{A}(x) + m] \int_0^\infty ds \exp[-is(m^2 - i0)] \times$$
$$\times \int DB\, \delta(x - y - 2\int_0^s d\xi B(\xi)) \exp\{-i\int_0^s d\xi [B^2(\xi) - \tag{13}$$
$$- g(2B_\mu(\xi) + \sigma_{\mu\nu}(\xi) i\hat{\partial}_x(\xi)) A_\mu(x - 2\int_\xi^s d\eta B(\eta))]\},$$

with the normalization of the measure DB: $\int dB \exp[-i\int_0^s d\xi B^2(\xi)] = 1$.

Neglecting the effects of the vacuum polarization in the infrared limit and the term $\sigma_{\mu\nu}$ in (13) we find for $G(x, y)$

$$G(x, y) = i\int_0^\infty ds \exp[-is(m^2 - i0)] \int DB \exp[-i\int_0^s d\xi B^2(\xi)] \times$$
$$\times [i\hat{\partial}_x + m + g^2 K(x, y|B)\, \delta(x - y - 2\int_0^s d\eta B(\eta)) \times \tag{14}$$
$$\times \exp[\frac{i}{2} g^2 \Phi(x, y|B)],$$

where

$$K(x, y|B) = \int_x^y d\xi^\nu \gamma^\mu D_{\mu\nu}(x - \xi) + 2\int_0^s d\xi \gamma^\mu D_{\mu\nu}[2\int_\xi^s d\eta B(\eta)] B^\nu(\xi),$$

$$\Phi(x, y|B) = 4\int_0^s d\xi_1 \int_0^s d\xi_2 B_\mu(\xi_1) D_{\mu\nu}[2\int_{\xi_1}^{\xi_2} d\eta B(\eta)] B^\nu(\xi_2) +$$

$$+ \int d\xi_1^{\mu} d\xi_2^{\nu} D_{\mu\nu}(\xi_1 - \xi_2) + 2\int_x^y d\xi^{\mu} \int_0^s d\xi B^{\nu}(\xi) D_{\mu\nu}[\xi - x + 2\int_{\xi}^s d\eta B(\eta)] +$$

$$+ 2\int_0^s d\xi \int_x^y d\xi^{\nu} B^{\mu}(\xi) D_{\mu\nu}[x - 2\int_{\xi}^s d\eta B(\eta) - \xi].$$

By passing to the momentum representation and performing a shift of the functional argument $B_{\mu}(\eta) = P_{\mu} + \omega_{\mu}(v)$, $\eta = sv$ we find from (14) that

$$G(p) = i\int_0^{\infty} ds \exp[is(p^2 - m^2 + i0)] \int D\omega \exp[-is\int_0^1 dv\,\omega^2(v)] \times$$

$$\times [\hat{p} + m + g^2 \tilde{K}(p|\omega)] \exp[\frac{i}{2} g^2 \Phi(p|\omega)]. \qquad (15)$$

In the infrared limit it is possible to neglect the functional argument ω in $\tilde{K}(p|\omega)$ and $\Phi(p|\omega)$. The functions $\tilde{K}(p|0)$ and $\Phi(p|0)$ are equal to zero as it is easy to see. Then from (16) we find that $G(p) \simeq (\hat{p} + m)|(m^2 - p^2)$ that completely agrees with (11).

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